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Comparing SVARs and SEMs: More Shocking Stories

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SOM-theme C: Coordination and growth in economies

Abstract

The structural vector autoregression (SVAR) and simultaneous equation macroeconometric model (SEM) styles of empirical macroeconomic modelling are compared and contrasted, with reference to two models of the UK economy, namely the Cambridge long-run structural VAR model and the COMPACT model. Various styles of impulse response analysis are also compared and contrasted, and used to illustrate model properties. The subtitle is a reference to the article "Shocking stories" by Levchenkova, Pagan and Robertson; in particular, their "reverse engineering" procedure is used to infer long-run relations of COMPACT comparable to the CSVAR cointegrating relations.

Keywords: cointegration, impulse response analysis, macroeconometric modelling, simultaneous equation models, structural VAR models, model comparisons
JEL-code: C51, C52

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1. Introduction

Macroeconometric models of national economies and the global economy continuously develop and evolve, and different styles of model have emerged during this process. The oldest is the simultaneous equation macroeconometric model (SEM) sometimes associated with the names of Tinbergen and Klein, who inaugurated this style of analysis in the 1930s and 1940s, although there has been substantial development since that time (see, for example, Wallis, 1995, 2000). Sometimes this model is referred to as a structural econometric model, using the first adjective in the traditional Cowles Commission sense. This was the focus of the critique by Sims (1980), who argued that the structural identification of such models is incredible. He proposed the alternative strategy of estimating unrestricted reduced forms treating all variables as endogenous, namely vector autoregressive (VAR) models. The subsequent recognition that, for policy analysis, VAR models still require identifying assumptions (Cooley and LeRoy, 1985, for example) resulted in a variety of ways of formulating such “structural VAR” (SVAR) models, starting from Bernanke (1986), Blanchard and Watson (1986) and Sims (1986) himself. In the meantime the cointegration literature that followed from Granger (1981) saw the VAR transformed into the vector error correction model (VECM), and an alternative proposal is to place identifying restrictions drawn from economic theory on the cointegrating relationships rather than the error covariances (see Pesaran and Pesaran, 1997, and references therein). This “long-run structural modelling” approach has recently been applied in the construction of a small quarterly model of the UK economy (Garratt *et al.*, 2000, 2001). The objective of the present paper is to compare this new model with a more conventional SEM, specifically the COMPACT model of the UK economy (Wren-Lewis *et al.*, 1996; Darby *et al.*, 1999). Since the new model is the result of a research project based at Cambridge University, we refer to it as the Cambridge structural VAR model, or CSVAR.

This paper’s subtitle is a reference to the article “Shocking stories” (Levtchenkova, Pagan and Robertson, 1998), some of whose ideas are pursued below. These authors bring together the cointegration literature and the empirical macroeconomics literature on the effects of shocks, and study the way in which information is brought to bear upon (stories are told about) impulse responses with respect to permanent and transitory shocks. Their illustration compares the small cointegrated VAR model of King, Plosser, Stock and Watson (1991) (KPSW) with the more theoretically based, global-economy MSG2 model (see McKibbin and Sachs, 1991).

Comparative analysis of macroeconometric models of the UK economy was undertaken on a systematic basis over the period 1983-99 by the ESRC Macroeconomic Modelling Bureau, a unique model-comparison project directed by the second author. The Bureau’s research programme included regular comparative studies based on standard simulation experiments, the last of which (Church *et al.*, 2000) also includes a review of the Bureau’s research methods and experience. All the models were SEMs; some were ever-present in the Bureau’s portfolio, while there were also new entries, including the COMPACT model from 1993 onwards, and exits. The present project considers the issues involved in comparing two different styles of model, and so represents an extension of the Bureau’s

research programme.

The paper proceeds as follows. Section 2 briefly reviews the different dynamic systems under consideration, their dynamic multipliers and impulse responses, and the permanent-transitory distinctions that arise once variables are allowed to be integrated. The generalized impulse response analysis proposed by Koop *et al.* (1996) and Pesaran and Shin (1998) is noted, since this approach is used by Garratt *et al.* (2000, 2001) in reporting the dynamic properties of the CSVAR model. Section 3 summarizes the key features of the CSVAR and COMPACT models, and considers various comparative questions. Section 4 presents the results of model comparisons in two specific dimensions, namely the different impulse responses to the shocks considered by the CSVAR modellers, which we also implement on COMPACT, and the models' cointegration properties, explicit or implicit. Section 5 concludes.

2. Dynamic systems: representations and properties

2.1 Simultaneous equations models

The dynamic linear SEM relating a vector of endogenous variables y_t to exogenous variables x_t with errors u_t is written in *structural form* as

$$B(L)y_t = G(L)x_t + u_t \quad (1)$$

where $B(L)$ and $G(L)$ are matrix polynomials in the lag operator L . The leading coefficient matrix B_0 is subject to some normalization rule and allows contemporaneous interactions among endogenous variables. These are removed in the *reduced form*, which expresses each endogenous variable as a function of predetermined variables, as follows,

$$y_t = B_0^{-1} \{ B_1 y_{t-1} + \dots + B_p y_{t-p} + G(L)x_t \} + B_0^{-1} u_t \quad (2)$$

with the convention that the autoregressive matrix polynomial is defined as

$$B(L) = B_0 - B_1 L - \dots - B_p L^p.$$

Under an appropriate stability condition on $B(L)$ the *final form* is given as

$$y_t = B(L)^{-1} G(L)x_t + B(L)^{-1} u_t. \quad (3)$$

Here each endogenous variable is expressed as an infinite distributed lag of the exogenous variables, together with an error term comprising moving averages of the structural errors. The coefficients in the expansion of $B(L)^{-1} G(L)$ are the *dynamic multipliers* or *impulse responses*, each column describing the effects on the endogenous variables over time of a unit shock to the corresponding exogenous variable. Cumulating these gives the *long-run multipliers* $B(1)^{-1} G(1)$, describing the steady-state responses to a unit step change or permanent shock in an exogenous variable. Similar impulse responses to direct perturbations to endogenous variables are given by the second term on the right-hand side of (3); SEMs are often programmed to allow such residual adjustments/intercept adjustments/add factors to elements of u_t in practical forecasting exercises. A further practical note is that

SEMs typically comprise mixtures of linear and log-linear equations and possibly include more complex forms such as the CES function, together with variables defined as products or ratios of other variables. Then the dynamic responses described above are calculated numerically, as the difference between two solutions of a model, one a base run or control solution and the second a perturbed solution in which the relevant shock is imposed. The study of steady-state or long-run implications then requires a sufficiently long simulation base.

2.2 Vector autoregressions

The VAR system is written

$$A(L)y_t = e_t, \quad (4)$$

where the leading matrix in the polynomial $A(L)$ is the identity matrix, reflecting the reduced form nature of the system. All n variables under consideration are treated as endogenous, thus (4) could be thought of as a “closed” version of (2). Impulse responses are calculated from the vector moving average representation

$$y_t = A(L)^{-1}e_t = C(L)e_t, \quad (5)$$

where the leading matrix in $C(L)$ is again the identity matrix. The elements of e_t are correlated, that is, $E(e_t e_t') = \Omega$ is not diagonal, and Sims (1980) argued that it is useful to transform them to orthogonal form to be able to see the “distinct patterns of movement” of the system. The triangular factorization $\Omega = T \Sigma T'$ where T is lower triangular with unit diagonal and Σ is diagonal gives the transformation $e_t = T \varepsilon_t$ such that $E(\varepsilon_t \varepsilon_t') = \Sigma$. The orthogonalized impulse responses $C(L)T$ then describe the consequences for y_{t+s} , $s = 0, 1, \dots$, of unit shocks to the individual, mutually uncorrelated elements of ε_t . For the last element these are the same as the original impulse responses, since the last column of T is $(0, \dots, 0, 1)'$; for all other elements the shock has an instantaneous impact, not only on the corresponding y -variable, as in the original system, but also on all variables placed lower in the y -vector. The orthogonalized impulse responses thus depend on the ordering of the variables in the VAR. Often a further scaling of these impulse responses is reported by considering the diagonal matrix $\Sigma^{\frac{1}{2}}$ of the standard deviations of ε_t . Defining $S = T \Sigma^{\frac{1}{2}}$ gives the Cholesky decomposition $\Omega = S S'$ and associated transformation $e_t = S \varepsilon_t^*$, and impulse responses to unit shocks to ε_t^* are then reported. These are $C(L)T \Sigma^{\frac{1}{2}}$, and describe the dynamic consequences for the y -variables of a shock of one standard deviation in the orthogonalized residuals. It is not clear how interpretability is improved by scaling in inverse proportion to the goodness of fit of the equations of the VAR.

Writing the orthogonalized VAR as

$$T^{-1}A(L)y_t = \varepsilon_t \quad (6)$$

gives the appearance of the Wold causal chain, with contemporaneous coefficient matrix that is lower triangular with unit diagonal, and uncorrelated disturbances. This arises from the orthogonalization procedure rather than the imposition of prior restrictions from relevant economic theory. The recognition that structural analysis in VAR models requires

such prior restrictions led to the development of SVAR models, as noted in the Introduction. The shocks are often given “structural” names, such as supply, money demand, technology, and so forth. Taking these to be the disturbance terms u_t of a closed version of the SEM (1), (2), with covariance matrix Σ , attention usually focuses on the relation

$$B_0 \Omega B_0' = \Sigma$$

and seeks restrictions that identify B_0 and Σ given the reduced form/VAR covariances Ω . This approach eschews restrictions on the dynamics, although in some applications long-run restrictions are used. It is common in the SVAR literature to assume Σ diagonal, but this is not done in the SEM literature, and whether it is a reasonable restriction on an SVAR has been questioned, by Bernanke (1986, pp.51-55) himself and Shiller in discussion of Blanchard and Watson (1986), for example, and in more recent reviews such as Pesaran and Smith (1998).

Generalized impulse response analysis (Koop *et al.*, 1996; Pesaran and Shin, 1998; for a precursor see also Evans and Wells, 1983) is an alternative to orthogonalization, whether this is the result of prior restrictions or simple renormalization. Rather than attempting to describe responses to specified shocks, generalized impulse responses (GIRs) describe the effect of “realistic” shocks, meaning shocks of the type that are typically or at least historically observed, as described by the sample estimate of the covariance matrix Ω . If this is not diagonal, a shock to one error is associated historically with changes in the other errors. The GIRs, defined as conditional expectations given the estimated system, describe its dynamic responses to the resulting composite or generalized impulse. They are given as $C(L)\Omega^\dagger$, where Ω^\dagger denotes the matrix obtained from Ω by dividing the elements of each column by its diagonal element, since $E(e_t | e_{jt} = 1) = (\omega_{1j}/\omega_{jj}, \omega_{2j}/\omega_{jj}, \dots, \omega_{nj}/\omega_{jj})'$. The GIRs are invariant to the ordering of the variables in the VAR, and coincide with the orthogonalized impulse responses for shocks to the first variable in the VAR, since when $j = 1$ the above column vector coincides with the first column of the matrix T defined above.

2.3 Cointegration and VECMs

The VAR system (4) can be rearranged as

$$A^*(L)\Delta y_t = -\Pi y_{t-1} + e_t$$

where $\Pi = A(1)$ and the degree of $A^*(L)$ is one less than that of $A(L)$. If the elements of y_t are $I(1)$ and cointegrated with $\text{rank}(\Pi) = r$, $0 < r < n$, then $\Pi = \alpha\beta'$ where α and β are $n \times r$ matrices of rank r , giving the VECM representation

$$A^*(L)\Delta y_t = -\alpha\beta' y_{t-1} + e_t. \quad (7)$$

Exact identification of β requires r restrictions on each of the r cointegrating vectors (columns of β): typically one is a normalization restriction. In the Wold representation of the difference-stationary variables

$$\Delta y_t = D(L)e_t \quad (8)$$

the matrix $D(1)$ of rank $n - r$ is given in Johansen's (1991) presentation of the Granger Representation Theorem as

$$D(1) = \beta_{\perp} [\alpha'_{\perp} A^*(1) \beta_{\perp}]^{-1} \alpha'_{\perp} \quad (9)$$

where the orthogonal complements α_{\perp} and β_{\perp} are $n \times (n - r)$ matrices of rank $n - r$ such that $\alpha' \alpha_{\perp} = 0$ and $\beta' \beta_{\perp} = 0$.

Various permanent-transitory decompositions follow from this representation. Stock and Watson (1988) show that, with r stationary linear combinations $\beta' y_t$, the $I(1)$ characteristics of y_t are given by $n - r$ "common trends" $\beta'_{\perp} y_t$. The shocks that drive the common stochastic trends are the shocks $\alpha'_{\perp} e_t$, called permanent shocks, leaving r transitory shocks: since $\beta' D(1) = 0$, shocks to the cointegrating vectors have no permanent effects. Writing (8) as

$$\Delta y_t = D(L) H^{-1} H e_t,$$

Levtchenkova *et al.* (1998) define a basic permanent-transitory decomposition as $H e_t$, with the first $n - r$ elements permanent and the last r elements transitory, that is, $D(1) H^{-1}$ has its last r columns equal to zero. Then H has the form

$$H = \begin{bmatrix} \alpha'_{\perp} \\ \rho' \end{bmatrix}$$

for any $n \times r$ matrix ρ such that H is invertible, and Levtchenkova *et al.* discuss various possible choices of ρ .

Despite identification of the cointegrating vectors by restrictions on β , permanent-transitory decompositions require further structural identifying restrictions, or stories. Given β and an initial choice of β_{\perp} , note that $\beta' \beta_{\perp} P = 0$ for any nonsingular $(n - r) \times (n - r)$ matrix P . If $n - r = 1$, then only a normalization restriction is required, as in KPSW's three-variable model, but if $n - r > 1$, then identification of individual common trends $\beta'_{\perp} y_t$ requires restrictions on β_{\perp} that make transformations $P' \beta'_{\perp} y_t$ inadmissible. Likewise, identifying individual permanent shocks requires further restrictions: Fisher *et al.* (2000), for example, provide an alternative identification of permanent shocks to that assumed by KPSW for their six-variable model. Finally, given an estimate of $D(1)$ of rank $n - r$ by appropriately shocking a model, its implied cointegrating vectors can be obtained by prescribing r^2 (just-identifying) elements of β and solving $\beta' D(1) = 0$ for the remainder. Levtchenkova *et al.* use such "reverse engineering" to back out the cointegrating vectors underlying the MSG2 model for comparison with those of the KPSW six-variable model, and this procedure is used in Section 4 below.

3. The CSVAR and COMPACT models

3.1 The Cambridge long-run structural VAR model

The CSVAR model incorporates long-run structural relationships suggested by economic theory as the cointegrating relations of a VECM. This is in reduced form, with short-

run dynamics restricted only by the choice of maximum lag. The long-run relations are based on production, arbitrage, solvency and portfolio balance conditions, together with stock-flow and accounting identities (Garratt *et al.*, 2001, §2). This approach is applied to a quarterly model of the UK economy, a small open economy. Since VAR modelling is restricted by the number of variables that can be dealt with, all of which are treated as endogenous, the number of variables describing each of the domestic and foreign economies is relatively small. The basic variables considered are domestic and foreign real per capita outputs (y, y^*), producer prices (p, p^*) and nominal interest rates (r, r^*), the nominal effective exchange rate (e), the price of oil (p^o) and the domestic real per capita stock of money (h), all in logarithms.

In the first version of the model (Garratt *et al.*, 2000), estimated over 1965q1-1995q4, the number of variables is reduced to eight, all treated as $I(1)$ for the purpose of cointegration analysis, by measuring domestic and foreign price variables relative to the oil price. There are five cointegrating relations, namely a purchasing power parity relation modified by the effect of oil prices (cf. Chaudhuri and Daniel, 1998); a nominal interest rate parity relation; an “output gap” relation based on common technological progress in production at home and overseas; and trade balance and real money balance relations based on long-run solvency conditions, these last two also containing time trends. With the variables ordered

$$(p - p^o, e, r, r^*, y, y^*, h - y, p^* - p^o)$$

the estimate of the 8×5 matrix β is

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -11.55 & 20.32 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1.52 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -0.91 & 0 & 0 & -1 & 0 \end{bmatrix}$$

This is highly overidentified, and the overidentifying restrictions are not rejected at the 10% level once bootstrapped rather than asymptotic critical values are compared to the likelihood ratio statistic. The foreign price relative to oil is then treated as weakly exogenous in the VECM, which thus has seven equations (and one lag); the econometric approach is described in Pesaran, Shin and Smith (2000).

The dynamics of the seven-equation VAR can be summarized by the 7×7 matrix $C(L)$ of impulse responses defined in equation (5), together with the responses of the endogenous variables to the weakly exogenous ($p^* - p^o$). These are presented diagrammatically in Figure 1. The innovations are one per cent (one annual percentage point for the interest rates), except for the foreign price relative to oil, which is scaled to give a long-run response of one per cent deviation from base. The innovations are applied in period 0, and responses are shown up to and including quarter 50, as in the GIR illustrations of Garratt

et al. (2000). Each column represents the responses to a shock to the diagonal element; off-diagonal impact effects for the first seven columns are equal to zero. The exogenous variable enters the VAR contemporaneously, although its impact effects, and dynamic effects more generally, are small for all but the domestic relative price variable. A temporary shock has a permanent effect on an $I(1)$ variable, and convergence to the long-run response is in most cases relatively quick. These are given alternatively as C_∞ or $D(1)$ in representations (5) and (8) respectively, and since $\beta'D(1) = 0$ the long-run responses of r and r^* are equal in each column, as are those of y and y^* .

The second version of the model (Garratt *et al.*, 2001) extends the estimation period to 1999q4 and adds the UK Retail Prices Index (\tilde{p}) to the list of variables. Now the price variables treated as $I(1)$ are the relative producer price levels $p - p^*$, the RPI inflation rate $\Delta\tilde{p}$ and the oil price p^o . The last is assumed to be “long-run forcing” but in the event does not appear in the cointegrating relations. Thus the PPP relation now takes its simple form, with no oil price effects. The cointegrating relations remain five in number, and the only other change is the replacement of the trade balance relation by a Fisher interest parity relation: the ex-post real interest rate is stationary. Again using bootstrapped critical values, the twenty-three overidentifying restrictions are not rejected at the 5% level ($LR = 71.49$). The two versions of the model are used in turn in Section 4 below.

3.2 The COMPACT model

The COMPACT model (Wren-Lewis *et al.*, 1996; Darby *et al.*, 1999) is an SEM, designed to translate modern macroeconomic theory into an econometric model that is both consistent with past evidence and capable of producing quantitative policy analysis; it is not used for forecasting. Its name reflects its relatively small size among UK SEMs, with fewer than twenty estimated behavioural equations, together with other estimated equations that set the weights in a disaggregated price system or estimate growth rates in simple projections, for oil output and population, for example. The model can be described as New Keynesian, combining traditional Keynesian nominal rigidities with forward-looking behaviour in wage and price setting. In the short run output depends on effective demand, the main components of which are separately modelled. In particular, the consumption function is consistent with a forward-looking Blanchard-Yaari life cycle model, except that some consumers are credit constrained and spend all their available income.

The model’s medium-term properties are strongly influenced by its vintage or “putty-clay” production technology in which there are no factor substitution possibilities once a new vintage of capital investment is installed, and so expected future factor prices are important in the investment decision. A change in technical progress, for example, is only gradually embodied in the capital stock as a result of its vintage structure. In this respect the model is unique among current UK SEMs. The only other UK SEM to have incorporated a vintage capital model of production in recent years is the National Institute’s domestic model (see Wren-Lewis, 1988), although this approach was abandoned in 1996 (see Church *et al.*, 1997, p.92). In the long run, however, the model behaves as if the ex-post (Cobb-Douglas) production technology were putty-putty. Its long-run properties are close to those of an

open-economy version of the standard neoclassical growth model: it is neutral, but not super-neutral, since nominal interest payments are taxed. In its normal operating environment the model has closure rules/reaction functions for fiscal and monetary policy. Further features of COMPACT are discussed in relevant contexts below.

3.3 Comparative issues

The CSVAR and COMPACT models clearly differ in size, the latter containing approximately ten times as many variables as the former. Comparisons of what the models have to say about the behaviour of key variables can only consider variables that are common to both, hence the greater detail that COMPACT provides must be left on one side. All but one of the variables of the CSVAR model have counterparts in COMPACT, sometimes with minor variation in the measure used, and this defines the range of our comparisons. The CSVAR variable that does not appear in COMPACT is money. It is obviously a component of the net financial assets variables that appear in COMPACT, but it is not separated out, and the model has no money demand function, for example. Thus our analysis of the cointegration implications of COMPACT focuses on four potential relationships and neglects the CSVAR money market equilibrium condition.

The VAR style of modelling is noticeably distinct from the SEM in abandoning the classification of variables as endogenous or exogenous. In the closed economy context of much of the early empirical VAR analysis—the US economy, that is—this meant treating policy variables as endogenous, and SEMs have tended to follow suit, now containing policy reaction functions in place of their previous treatment of policy instruments as exogenous. In an open economy context, however, the distinction remains, as in our present examples. The CSVAR model treats the overseas economy in the same way as the domestic economy, whereas COMPACT neglects the effect of the UK economy on the rest of the world, and treats “world” variables as exogenous. The world investment and output variables are related to the world trade variable to ensure mutual consistency of their growth paths, but this sub-system is causally prior to the rest of the model. There are no interactions in COMPACT among the three world variables treated as endogenous in CSVAR, hence related movements in these variables would need to be imposed on the COMPACT solution by the model user, as illustrated below.

Studies of the dynamic properties of the two systems proceed in different ways, as briefly noted above. The CSVAR model is linear, hence impulse responses can be calculated from expressions such as those in Section 2, or as the difference between a base run and a perturbed solution of the model, as in the nonlinear case. In presenting plots of dynamic responses, it is customary to use a time scale sufficiently long that convergence and steady-state questions can be visually assessed, as in the above example. Long-run responses can be calculated directly from expression (9) to check how close dynamic responses are to their steady-state values.

The COMPACT model is nonlinear, hence numerical methods are used, although nonlinearity is not a serious problem in standard simulation exercises: doubling the shock doubles the response, for typical perturbations. There is no formal description of the model's

long run, although its theoretical properties are well understood, hence a long simulation period is required to observe the long-run solution. The model treats forward expectations variables as rational or model-consistent expectations, and so it is important to ensure that the terminal date is sufficiently far into the future that the simulation is unaffected by this choice. These two considerations lead to the use of a solution period of 280 quarters in the model's normal mode of operation.

4. Comparative dynamics

4.1 Impulse responses to a foreign output shock

Our comparison of the different impulse responses uses the same shock as Garratt *et al.* (2000, Figs 5.2-5.4), namely a shock to the foreign output equation. Their generalized impulse responses (GIRs) relate to a composite shock to all variables of the model, as given by the correlations between the residuals of the foreign output equation and the other equations. We compare these to conventional impulse responses (IRs), in which only the foreign output variable is shocked, given in the sixth column of Figure 1. As noted above, the traditional approach of shocking a single variable without reference to possible residual correlations is equivalent to placing the variable in the last position when using the Cholesky orthogonalization.

The shocks are designed to give an increase in foreign output of 1% in the long run. To calibrate the required initial shocks and help interpret the long-run responses we first calculate the matrix $D(1)$ of long-run multipliers via equation (9) with the following results.

$$\begin{bmatrix} 1.089 & 0.664 & -2.306 & 4.645 & 0.247 & -0.058 & 0.358 & -0.702 \\ 1.089 & 0.664 & -2.306 & 4.645 & 0.247 & -0.058 & 0.358 & -5.281 \\ 0.028 & -0.005 & 0.095 & 0.016 & -0.001 & 0.022 & -0.030 & -0.153 \\ 0.028 & -0.005 & 0.095 & 0.016 & -0.001 & 0.022 & -0.030 & -0.153 \\ 0.211 & -0.035 & 0.722 & 0.125 & -0.007 & 0.166 & -0.299 & -0.880 \\ 0.211 & -0.035 & 0.722 & 0.125 & -0.007 & 0.166 & -0.299 & -0.880 \\ -0.565 & 0.093 & -1.937 & -0.334 & 0.018 & -0.446 & 0.614 & 3.116 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5.008 \end{bmatrix} \begin{matrix} p - p^o \\ e \\ r \\ r^* \\ y \\ y^* \\ h - y \\ p^* - p^o \end{matrix}$$

The variables are ordered as in Section 3.1, repeated in the right-hand column for convenience. Since there are five cointegrating relations among the eight variables, this matrix has rank 3. The first two rows are equal except for the last element, which is the only non-zero element in the last row, whose first seven zeros reflect the treatment of $p^* - p^o$ as weakly exogenous in the VECM and so not affected by shocks to the other variables. The third and fourth rows are equal, the long-run effect of shocks on the cointegrating combination $r - r^*$ being zero, likewise $y - y^*$, the fifth and sixth rows. Finally rows 3, 5 and 7 are proportional to one another (though rounding may obscure this).

To achieve a long-run IR of 1% in foreign output an impulse of $0.166^{-1} = 6\%$ is required. To achieve the same long-run GIR of foreign output the required impulses to the seven

variables depend on the long-run responses of foreign output to the individual shocks, as given by the sixth row of $D(1)$, and their correlations with the foreign output shock. In the event the combined shock's foreign output impulse is 2.5%. The complete set of shocks that form the generalized impulse is

$$\begin{array}{ccccccc} p - p^o & e & r & r^* & y & y^* & h - y \\ -0.36 & -0.81 & 0.27 & 0.19 & 1.80 & 2.52 & -1.84 \end{array}$$

and using these weights to construct a linear combination of the first seven columns of Figure 1 yields the GIRs. Since the long-run responses of the third through seventh variable to the different shocks are in the same proportion, it follows that the long-run IR equals the long-run GIR for each of these variables. This does not hold for the first two variables, since their responses to the various shocks differ from those that determine the weights in the combined shock that equates the long-run GIR and IR for foreign output. Nevertheless the long-run IRs of these variables are equal to one another, and so are their long-run GIRs in this experiment, since the foreign price variable is not perturbed.

The impulse responses are plotted in Figure 2. The time profiles of the GIRs correspond to those in Figures 5.2-5.4 of Garratt *et al.* (2000), though the scale is slightly different; the time horizon is the same. The horizon of 50 quarters is not quite enough to demonstrate the convergence of IRs and GIRs in all the cases identified in the previous paragraph. Running the solution on shows that this is accomplished in 75-100 quarters.

The GIRs show a non-zero initial impact in all variables, by design, and this combined shock brings foreign output to its long-run response much more quickly than the shock to this variable alone. The shock to $h - y$ makes approximately the same contribution to the 1% long-run response of foreign output as the y^* shock itself, and we note from the (6,7) cell of Figure 1 that the foreign output response to $h - y$ reaches its long-run level relatively quickly. For other variables the IRs show greater fluctuations over the first 2-3 years than the GIRs, the swing in the inflation rate being particularly noticeable. Thus in general, averaging the IRs across the rows of Figure 1 to obtain the GIRs serves to smooth the intermediate fluctuations.

Similar combinations of IRs yield orthogonalized impulse responses, once an ordering of variables has been chosen and a corresponding triangular factorization or Cholesky decomposition of Ω undertaken. In a cointegrated VAR in which IRs do not converge to zero, this example illustrates that different combinations imply different long-run responses and hence that the choice of ordering is not innocuous, contrary to what is often asserted by proponents of orthogonalization, who perhaps have stationary or trend-stationary systems in mind.

Two impulse responses for COMPACT are also plotted in Figure 2. The first, labelled YS, relates to a shock to the world output, investment and trade variables, which are constrained to move in step, but to no other exogenous variables. The domestic responses to this shock are small. The nominal exchange rate jumps on impact, then slowly declines to reach a steady-state level 0.21% below base. This is close to the long-run OIR of the exchange rate in CSVAR, which is 0.35% below base, but these experiments are not as comparable

as they appear at first sight, as is discussed next.

The design of our second COMPACT experiment is motivated by the “output gap” cointegrating relation of CSVAR, which equates real per capita output levels at home and overseas. This equation is derived from a stochastic version of the Solow growth model in which there is common technological progress in domestic and foreign production. Thus we interpret a foreign output shock as a global technology shock and, in accordance, also adjust domestic technical progress in COMPACT. (Since the modelled variable is non-oil GDP, and the UK output of oil is simply projected forward at a constant growth rate, for consistency we also adjust this growth path.) The responses are labelled GS in Figure 2. The solution period for COMPACT is 280 quarters, as noted above, and the rate of convergence is slow, so that after the 50 quarters shown in the plots considerable adjustment still remains to be accomplished. The eventual steady-state increase in domestic output is 1%, as in CSVAR. The domestic interest rate returns to base, and as the foreign interest rate has not been perturbed and the uncovered interest parity condition holds, the exchange rate stabilizes, at a level 3.4% below base. Among other variables employment returns to base, so that the increase in average labour productivity matches the increase in technical progress and the natural rate of unemployment is unaffected, although there are transient unemployment costs during the adjustment process.

The relatively sluggish response of domestic output in COMPACT to improved technology is due to its vintage capital production system, in which it takes approximately 20 years before all the capital stock in use benefits from the improvement. The CSVAR model does not explicitly model the investment process and the influence of technical progress upon it, but simply captures the “stylized facts” of the dynamic interrelationships of domestic and foreign output and the remaining variables of the system. To the extent that the technical progress “story” behind the foreign output shock in CSVAR is treated as comparable to an intervention on COMPACT’s rate of labour-augmenting technical progress, the VAR evidence suggests that the domestic output response in COMPACT is unrealistically slow. Although this is the only UK SEM that incorporates a vintage capital model of production, as noted above, Gilchrist and Williams (2000) argue for its empirical relevance in the US. Their DSGE model with putty-clay technology shows much quicker output responses to technology shocks than COMPACT, suggesting that dynamic specification choices within the vintage framework, rather than the vintage framework itself, may be the cause of COMPACT’s sluggishness.

4.2 Effects of an oil price shock

Our second simulation reproduces the oil price experiment of Garratt *et al.* (2001). This uses the second version of CSVAR, in which the oil price variable appears in its own right, as an exogenous variable in the model’s short-run dynamics. The shock is an increase of 16.485%, equal to one standard error of the projection equation for this variable which is estimated for use in other model exercises.

Impulse responses for six variables of interest are shown in Figure 3, labelled CAM2. These correspond to results shown in the relevant panels of Garratt *et al.* (2001)’s Figure

2, where 95% confidence error bands are also shown. We note that, in contrast to our deterministic simulation results, they show the mean values of the empirical distributions of the impulse responses generated from the bootstrap procedure used to calculate the standard error bands, but there are no perceptible differences between the two sets of results. Their confidence intervals cover zero for all but a few early periods for a few variables, although the authors note their lack of reliability as indicators of the precision of the estimates beyond the first one or two years.

Comparisons with COMPACT again raise the question of the different treatment of variables as endogenous or exogenous in the two models, although this is a simpler example than the foreign output shock because both models treat the oil price as an exogenous variable. It also turns out that the main comparison of interest is largely unaffected by assumptions about other exogenous variables in COMPACT. In the first set of COMPACT results included in Figure 3, labelled OIL, only the oil price variable is perturbed. In the second scenario, labelled OIL*, an exogenous change in world output equal to the long-run endogenous change in domestic output is also assumed, again motivated by the cointegration results of CSVAR, and oil output is perturbed, as in the GS scenario above. These changes have little effect on the domestic output response, however, as would be expected in the light of the results of the foreign output shock.

The domestic output response provides the major contrast between the two models. Aside from questions of its statistical significance, this is negative in CSVAR, approximately 0.24% below base after 2.5 years, and strongly positive in COMPACT in the medium to long run. Foreign output responds more slowly than domestic output in CSVAR, but reaches the same long-run decline of 0.16%. Garratt *et al.* (2001) describe the negative sign of the effect on output as “expected”, although there is no “story” provided and the oil price does not appear in the model’s theoretical development. In COMPACT the UK is a net oil exporter, and so receives a permanent income gain from a permanent increase in the oil price. This increase in income allows the economy to increase the capital stock, supporting permanently higher output, 0.5% above base in the long run. The rise in the oil price improves the current account, and balance is restored by increased import demand induced by the change in output.

The CSVAR model does not provide comparable detail, but simply reflects the correlations between its endogenous variables and the oil price in a reduced form manner. The behaviour of the price of oil over the sample period is dominated by four main shocks, namely the OPEC I and II rises of the 1970s, the sharp fall in 1985-6, and the rise associated with the Iraqi invasion of Kuwait in 1990. The first two were the more dramatic changes, each followed by recession. The subsequent consensus, however, is that each recession had more than this single cause. Monetarists point to the surge in monetary growth in 1972-3 which preceded the inflationary explosion and falling output of 1974-5, while other commodity prices than oil were also rising exceptionally quickly in 1973. The 1979 oil-price shock initiated a second recession across the OECD economies, although in the UK it started earlier than elsewhere and was of much greater severity, despite its near self-sufficiency in oil by that time. Subsequent analysis places greater weight on the re-

strictive monetary and fiscal policies of the June 1979 budget (Mrs Thatcher's first) and the unanticipated deterioration in competitiveness as factors explaining the shortfall in output. Disentangling these effects clearly requires a more elaborate system than an eight-variable VAR.

4.3 Long-run structural relations

To compare the long-run implications of COMPACT with the long-run structural relations of CSVAR, estimated as cointegrating relations, we employ the reverse engineering procedure of Levchenkova *et al.* (1998), discussed in Section 2.3. For this purpose we require an estimate of COMPACT's long-run responses to permanent shocks for variables comparable to those of CSVAR. As noted above, the money variable does not appear in COMPACT, so the number of variables is reduced to seven. In the CSVAR cointegrating relations this variable appears only in its "own" money market equilibrium condition, so removing this equation from consideration and focusing on the remaining four long-run relations causes no additional complication. For the present purpose we use the second version of CSVAR which, like COMPACT, treats the oil price as exogenous. The four cointegrating relations under consideration are heavily overidentified, the only estimated coefficient in each relation being the constant term. With relative prices $p - p^*$ defined as a single variable, the four cointegrating combinations of the seven variables are $p - p^* - e$, $r - r^*$, $y - y^*$ and $r - \Delta \tilde{p}$.

These adjustments leave the number of permanent shocks at three, and we design three COMPACT scenarios that have permanent effects. The first is the global technology shock of Section 4.1 in which a long-run increase in exogenous foreign output is assumed to be the result of an increase in global technological progress, which produces the same long-run response in endogenous domestic output; no other foreign variables are perturbed. The second is a monetary policy shock, in which the inflation target in the monetary policy rule is increased by 0.1%. The long-run outcome is an increase of the same amount in domestic inflation and domestic interest rates. The equivalent increases in exogenous foreign interest rates and inflation are imposed, although the terminal price levels differ due to different adjustment paths. The third shock is the oil price increase of Section 4.2, with an exogenous increase in world output equal to the endogenous increase in domestic output also assumed, as in the second variant of that experiment (OIL*).

The long-run responses of the seven relevant variables in COMPACT in these three scenarios are as follows:

$$\begin{bmatrix} 3.30 & 51.40 & -0.02 \\ 3.37 & 0.55 & -2.08 \\ 0 & 0.1 & 0 \\ 0 & 0.1 & 0 \\ 1 & 0 & 0.51 \\ 1 & 0 & 0.51 \\ 0 & 0.1 & 0 \end{bmatrix} \begin{matrix} p - p^* \\ e \\ r \\ r^* \\ y \\ y^* \\ \Delta \tilde{p} \end{matrix}$$

These three columns correspond to the nonzero columns of $D(1)H^{-1}$ in the basic permanent-transitory decomposition of Levchenkova *et al.* (1998) discussed above, and we seek their orthogonal complement such that $\beta'D(1)H^{-1} = 0$ for comparison with the cointegrating combinations of CSVAR. It is immediately clear that $r - r^*$, $y - y^*$ and $r - \Delta\tilde{p}$ appear here, as in CSVAR, whereas a PPP relation does not. Normalizing a fourth relation on $p - p^*$, and assuming nonzero coefficients on e , r and y , yields $p - p^* - 0.45e - 511r - 1.79y$, which deviates from the PPP relation of CSVAR. (It contains the same variables as the “trade balance” cointegrating relation in the first version of CSVAR described in Section 3.1, although the coefficients on the last three variables take different values and the complete sets of variables are not the same.)

Calculations analogous to those of Levchenkova *et al.* (1998, fn.15) show that replacing CSVAR’s PPP relation by the above relation (and retaining its fifth cointegrating monetary equilibrium relation to ensure comparability) increases the LR statistic from the value 71.49 noted above to 85.35. This reflects the different theoretical structure of COMPACT, which does not assume that PPP holds in the long run. In its small open-economy context, with the rest of the world exogenous, the real exchange rate in the long run equates the demand and supply of domestic output.

5. Conclusion

The two models of the UK economy studied in this paper each aim, within their different styles of modelling, to provide a sounder foundation in modern macroeconomic theory than is customary in that style. Comparisons between them, however, are limited more by particular features of each modelling style than by differences in their theoretical structures. Two features that strongly influence the nature of the analysis undertaken here are the “curse of dimensionality” insofar as it restricts the number of variables that VAR models can handle, and the different approaches to exogeneity questions, which have a similar impact on the size of the VAR and also influence the design of comparable scenarios. That policy variables should be treated as endogenous is generally accepted by both SVAR and SEM schools; in the context of models of small open economies, however, differences remain in the treatment of variables describing the external economic environment. If these are taken to be individually exogenous, unmodelled variables, then the design of global-economy scenarios in which such variables interact realistically requires off-model information, and there are obvious opportunities to combine information from the two approaches, as shown in the exercises presented above.

A further approach in which the external economic environment is endogenized is that of the multicountry econometric model, in which several countries or groups of countries are each modelled at a level of detail comparable to that of a national-economy SEM, and their trade and financial interrelationships are made explicit. Various spillover questions and responses to asymmetric shocks can be analyzed, but even in a two-country world the appropriate size of the model considerably exceeds that of the CSVAR model, which remains silent on such matters. The COMPACT results in our oil price simulation show

that whether or not the UK is a net oil exporter has an important bearing on the nature and interpretation of the response to an oil price shock, but the eight-variable CSVAR lacks the relevant information to take a position on this question.

Two aspects of the dynamic properties of the models are studied in this paper, namely impulse response analysis and cointegration analysis. Our experience with the former is that traditional shock-one-thing-at-a-time dynamic multiplier analysis remains an essential part of the toolkit. Even if the argument for generalized impulse response analysis - that this is how shocks were cross-correlated in the data - is accepted, understanding of the responses to a package of shocks requires knowledge of the individual component responses; here, as elsewhere in economics, these partial derivatives are the key to understanding the behaviour of the system. Focussing on the steady-state or long-run implications does not evade the issue: in a cointegrated VAR impulse responses converge to zero only for the cointegrating combinations of variables, and the individual responses are the basic ingredients of the analysis of the response of any other combination of variables, whether constructed by orthogonalization, generalization, etceterization.

Cointegration analysis and the error correction model are in widespread use in both small-scale and large-scale macroeconometric modelling, and their general principles are accepted by both SVAR and SEM modellers, as in our present examples. The curse of dimensionality leads the latter to use this approach in the specification of individual estimated equations, or small subgroups of related equations, within a larger system, whereas the former adopt a system-wide approach. Differences arise in statistical modelling, however, since the long-run or equilibrium relations of a SEM are rarely restricted to include only integrated variables, whereas this is a requirement of the VECM approach implemented in the CSVAR model. As its authors acknowledge, their treatment of interest rates and inflation rates as integrated variables raises interesting issues concerning the use of economic theory and statistical evidence in macroeconometric modelling, and the nature of the policy experiments that can be implemented on their model, and these are a subject of continuing research in the context of model comparisons.

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Figure 1 Impulse responses of the Cambridge structural VAR model

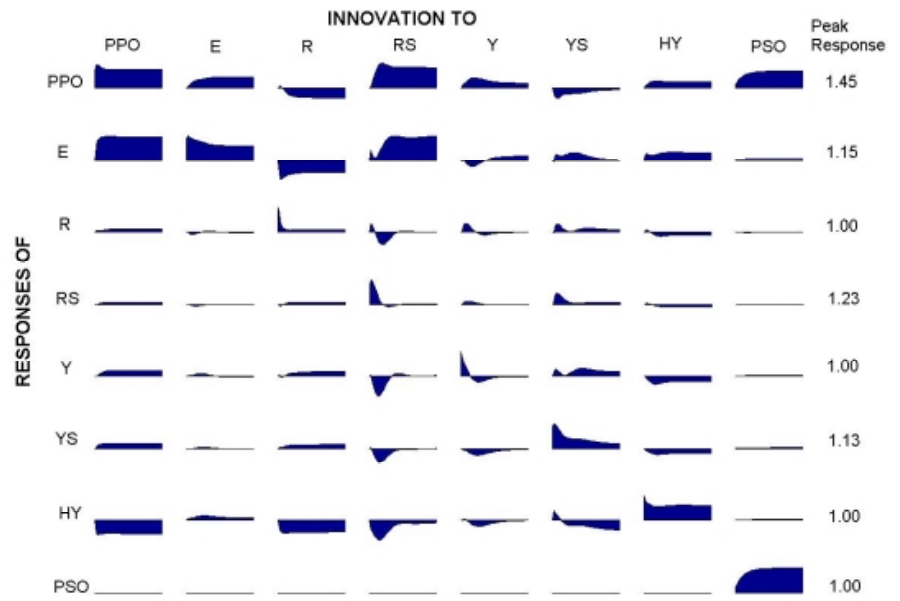


Figure 2 Impulse responses to a permanent 1% foreign output shock

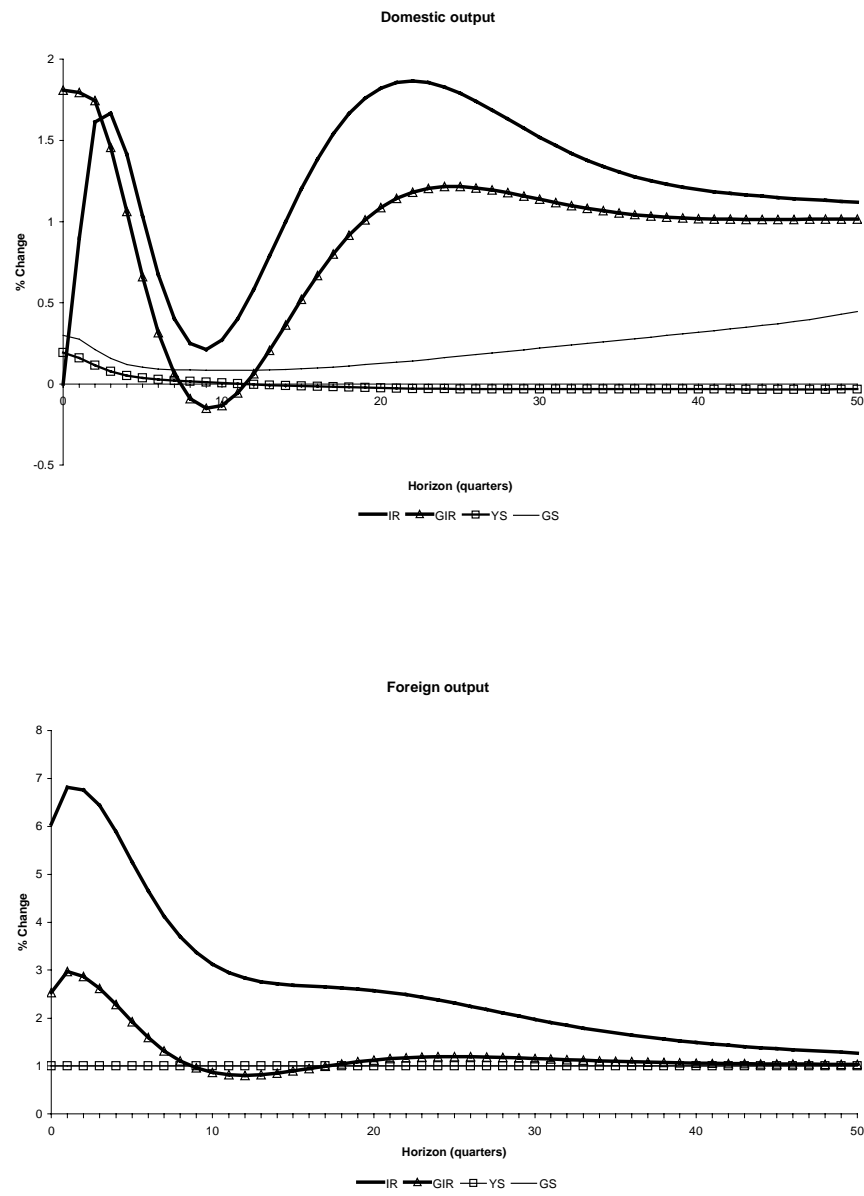


Figure 2 (continued)

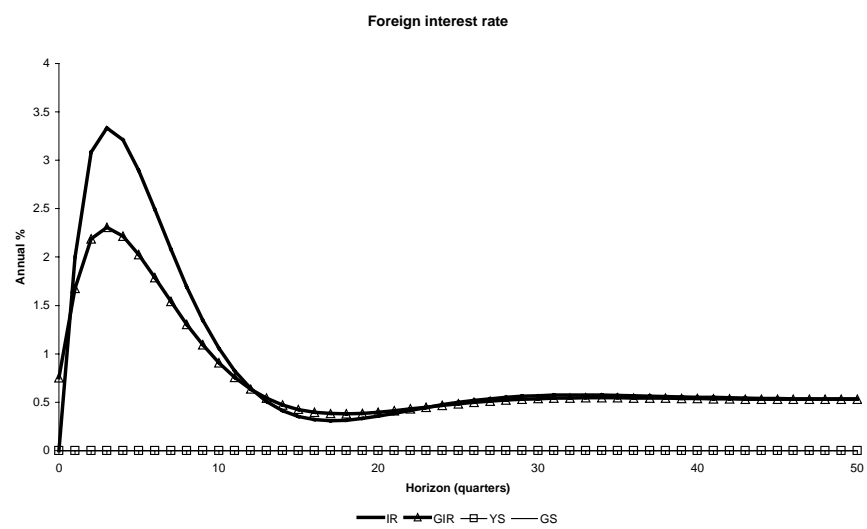
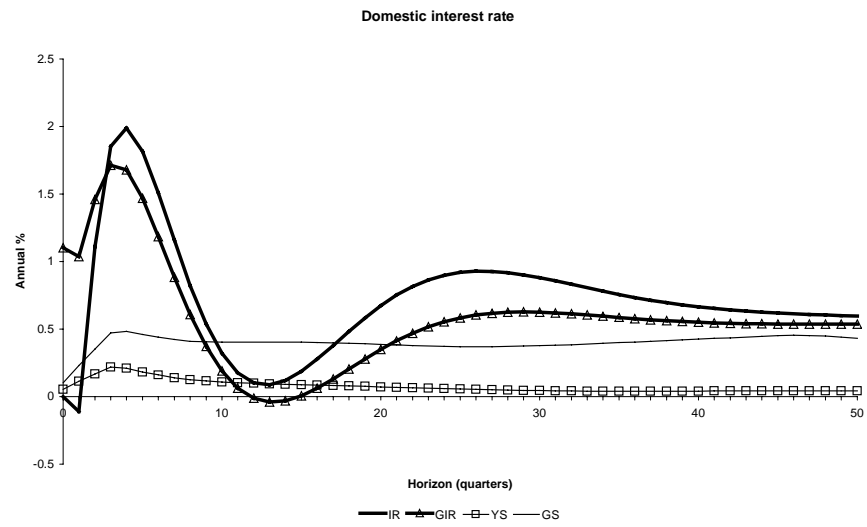


Figure 2 (concluded)

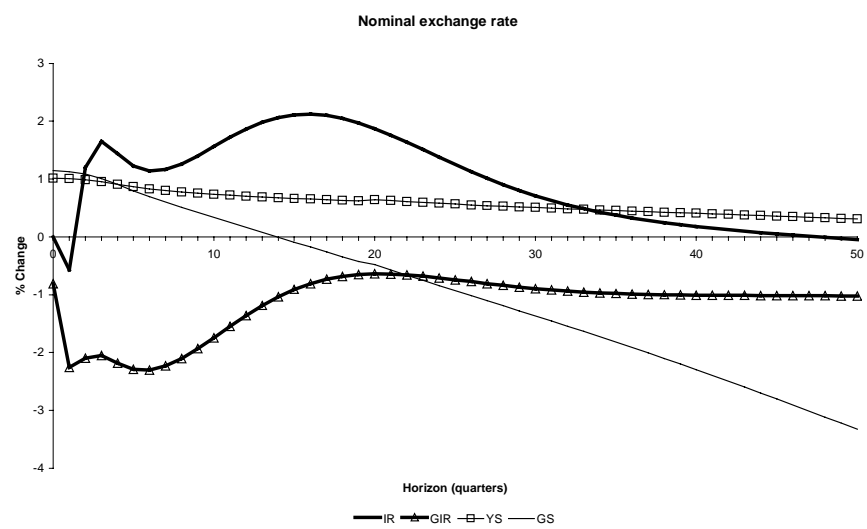
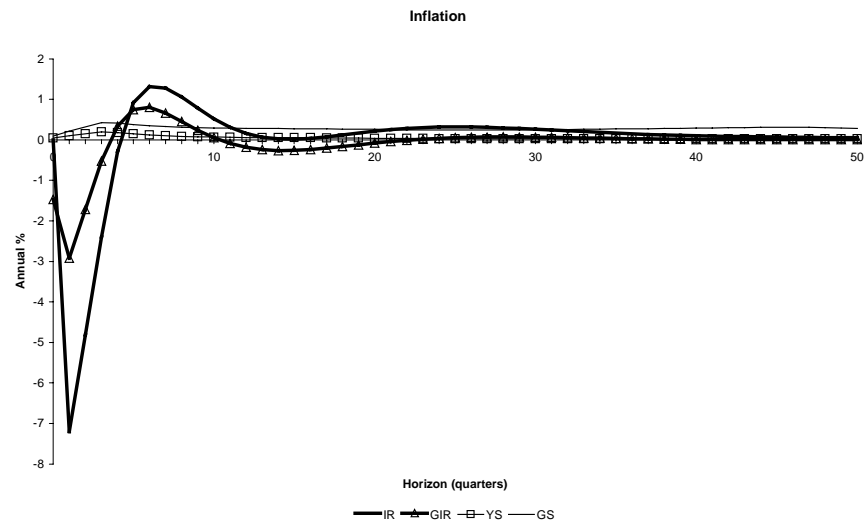


Figure 3 Impulse responses to a permanent 16.485% oil price shock

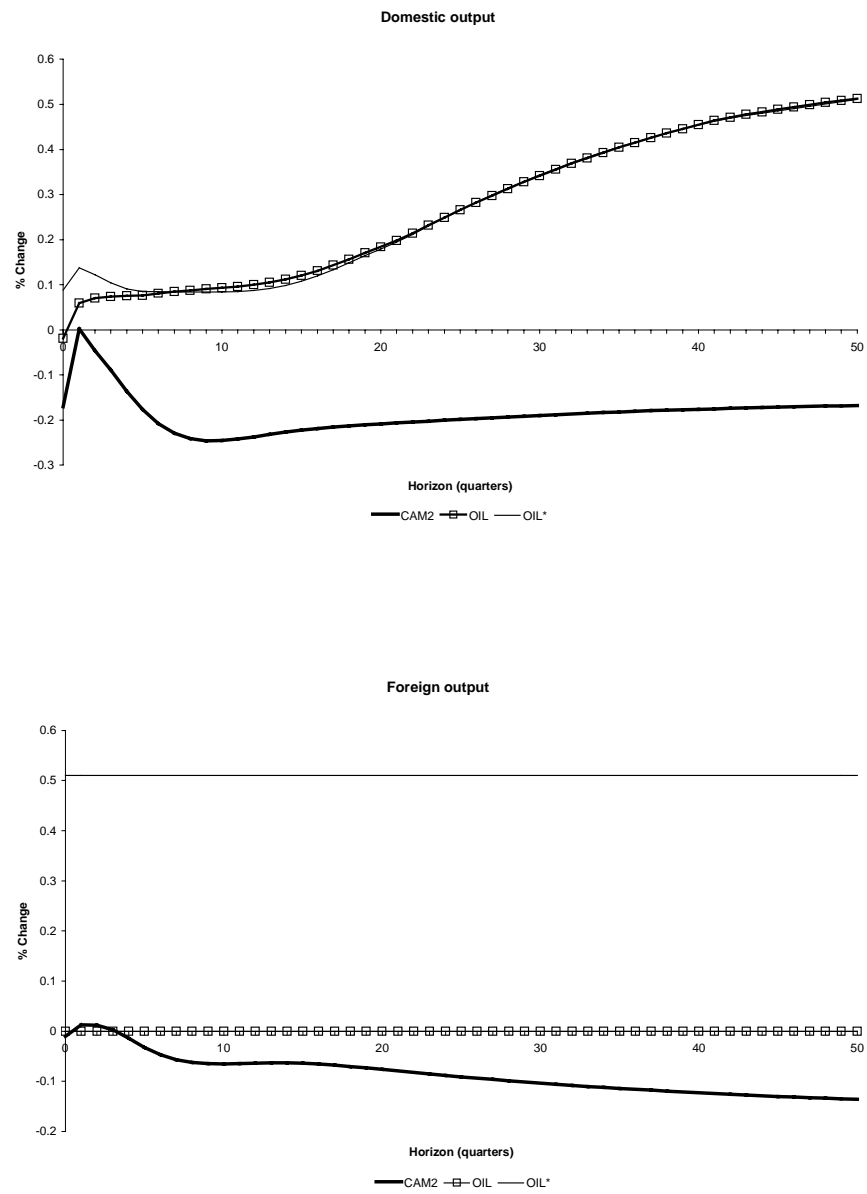


Figure 3 (continued)

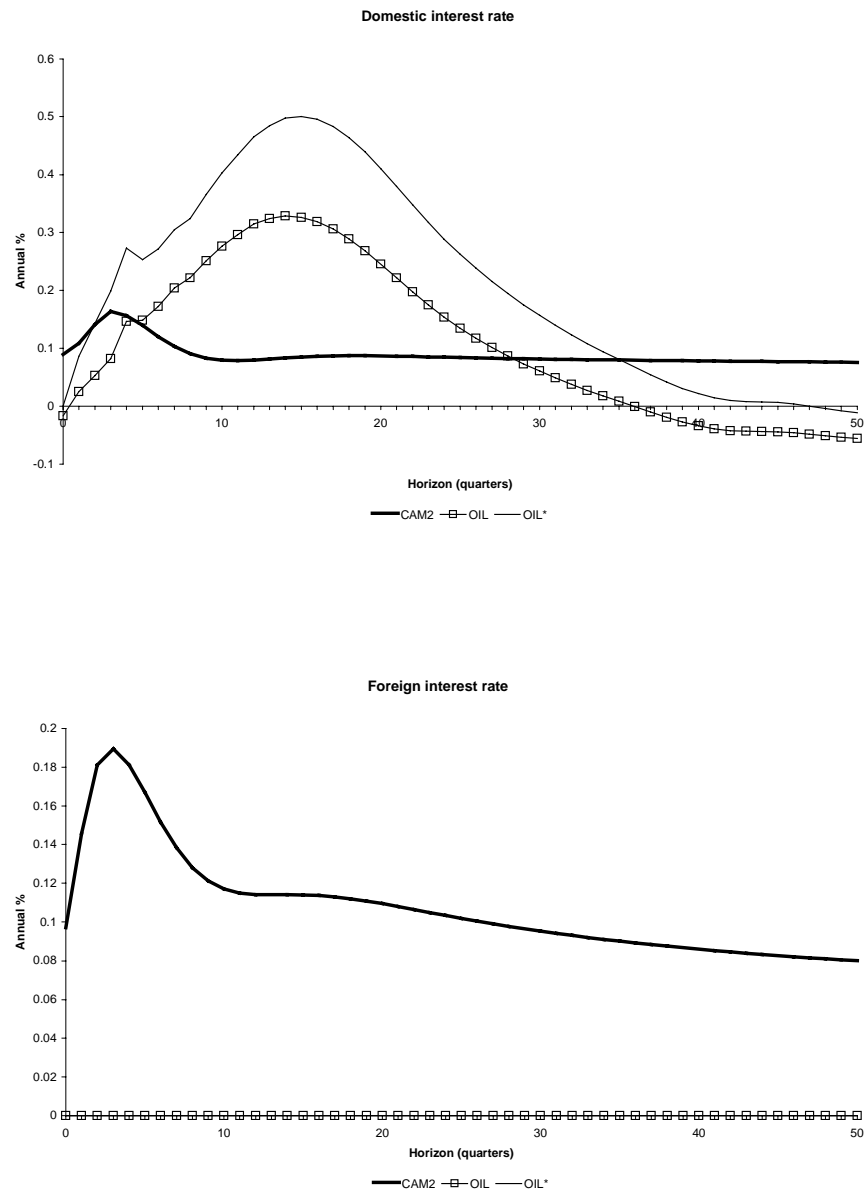


Figure 3 (concluded)

